A note on exclusion statistics parameter and Hausdorff dimension

Wellington da Cruz*
Departamento de Física,
Universidade Estadual de Londrina, Caixa Postal 6001,
Cep 86051-970 Londrina, PR, Brazil

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Abstract

We obtain for an anyon gas in the high temperature limit a relation between the exclusion statistics parameter g and the Hausdorff dimension h, given by $g = h(2-h)$. The anyonic excitations are classified into equivalence classes labeled by Hausdorff dimension, h, and in that limit, the parameter g give us the second virial coefficient for any statistics, ν. The anyonic excitations into the same class h get the same value of this virial coefficient.

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*E-mail: wdacruz@fisica.uel.br

We have obtained in [1] a distribution function for anyonic excitations classified into equivalence classes labeled by the fractal parameter h, the Hausdorff dimension. We have that for, $h = 2$ we get bosons and for $h = 1$ we get fermions. For, $1 < h < 2$, we have anyonic excitations which interpolates between these two extremes. The fractal parameter h is related to statistics ν, in the following way:

$$
\begin{array}{llll}
h-1=1-\nu, & 0<\nu<1; & h-1=\nu-1, & 1<\nu<2; \\
h-1=3-\nu, & 2<\nu<3; & h-1=\nu-3, & 3<\nu<4; \\
h-1=5-\nu, & 4<\nu<5; & h-1=\nu-5, & 5<\nu<6; \\
h-1=7-\nu, & 6<\nu<7; & h-1=\nu-7, & 7<\nu<8; \\
h-1=9-\nu, & 8<\nu<9; & h-1=\nu-9, & 9<\nu<10; \\
etc, & & &
\end{array} \tag{1}
$$

such that this spectrum of ν shows us a complete mirror symmetry. Now, we obtain a connection between the statistics ν and the exclusion statistics parameter g [2] for an anyon gas in the high temperature limit, as follows:

$$
\begin{array}{ll}
g=\nu(2-\nu), & 0<\nu<2; \\
g=(\nu-2)(4-\nu), & 2<\nu<4; \\
g=(\nu-4)(6-\nu), & 4<\nu<6; \\
g=(\nu-6)(8-\nu), & 6<\nu<8; \\
g=(\nu-8)(10-\nu), & 8<\nu<10; \\
etc. &
\end{array} \tag{2}
$$

We observe that in [3] g was obtained only for the first interval and this parameter is proportional, in general, to the dimensionless second virial coefficient if the equation of state of the system admits a virial expansion in the high temperature limit. In this note, we follow an approach completely distinct and extend that result putting it under a new perspective. The expressions Eq.(2) were possible because of the mirror symmetry as just have said above. On the other hand, our approach in terms of Hausdorff dimension, h [1] collect into equivalence class the anyonic excitations and so, we consider on equal footing the excitations in the class. Therefore, we have established independently of the approach given in [3] the relation between g and the second virial coefficient for the complete spectrum of ν, that is, we have found that the exclusion statistics parameter g is related to h, for an anyon gas in the high temperature limit, as follows

$$g = h(2-h). \tag{3}$$

We can check Eq.(3) using the relations between h and ν Eq.(1), obtaining the expressions Eq.(2). On the other hand, as the second virial coefficient, $\mathcal{B}_2(g)$, determines the exclusion statistics parameter [3], we have that

$$
\begin{aligned}
\mathcal{B}_2(g) &= \frac{g}{2} - \frac{1}{4}; \\
\tilde{\mathcal{B}}_2(h) &= \frac{h(2-h)}{2} - \frac{1}{4},
\end{aligned} \tag{4}
$$

where the second expression follows from our approach and it is more general, in the sense that, the second virial coefficient is written now for each class of the anyonic excitations. The expressions Eq.(4) have the correct values for bosons, $\mathcal{B}_2(0) = -\frac{1}{4} = \tilde{\mathcal{B}}_2(2)$ and for fermions, $\mathcal{B}_2(1) = \frac{1}{4} = \tilde{\mathcal{B}}_2(1)$. In this way, we see that the anyonic excitations into the class h have the same value for the second virial coeficient in the high temperature limit.

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